Synthesis of Full-Body 3-D Human Gait using Optimal Control Methods

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\textbf{Abstract}—In this paper we present a method that uses optimal control for offline human gait synthesis that does not depend on motion capture data or task-specific controllers. Our method is based on efficient simulation of rigid multibody systems and a direct multiple-shooting method to solve the underlying space-time optimization problem. We formulated different optimization criteria and synthesized gaits for a full-body 3-D human model with 34 degrees of freedom and compared the resulting movements with human data. By combining different criteria we are able to improve the similarity of the synthesized motions with respect to recorded human motion-capture motions.

\section{I. INTRODUCTION}

Human walking is a complex task that involves coordination of the whole body. While effortlessly performed by humans it remains a difficult challenge for humanoid robots. Gaining a better understanding of human movement is of great interest in many areas of robotics, for the design of better robots, orthoses and prostheses, and exoskeletons. But also other research domains such as biomechanics is interested in the study of human gait. Classical gait analysis is mostly concerned with post-hoc analysis of recorded motions, while gait synthesis methods enable predictive gait analysis and allow the study of human gait in a completely different manner.

In our research we use optimal control methods for both motion analysis and gait synthesis [1], however in this paper we focus on the latter for reasons of space. The goal of this paper is to present our latest results on gait synthesis that can be seen as an extension to our previous work presented in [2], which used offline optimization to synthesize walking motions in the sagittal plane. The contributions of this paper are:

- We describe a formulation of the human gait that is piecewise continuously differentiable and can thus be used in a optimal control context. We model the human using a rigid multibody model and use discrete constraint sets for the different contact configurations of the foot with the ground. Contact gains are treated as instantaneous inelastic collisions.
- We present a optimal control formulation that we use to synthesize human-like gait. The physical simulation of the human model is posed as a constraint of the problem and task-space variables such as walk velocity and step length can be prescribed to yield corresponding movements. We explicitly formulate periodicity to obtain cyclic motions, however our approach can be extended to produce non-cyclic motions.
- We use different objective functions to synthesize walking animations and compare those with recorded motion capture data.

\section{II. RELATED WORK}

Synthesis of human walking motions has been the focus of many research efforts over the last two decades. A wide range of models and approaches of creating believable or even physically feasible motions exists. Models inspired from biomechanics range from simple inverted pendulums [3] to complex multibody systems with simulated muscles [4], [5]. In computer graphics a lot of work is being done to synthesize believable motions. Some of them use existing motions and optimization techniques to compute or find a transition from one motion to another [6], [7].

Other works in this area incorporate dynamics simulation to generate believable motions. In [8] a dynamical model and optimal control methods are used to animate a lamp figure with six degrees of freedom. This so-called spacetime constraint approach uses a two boundary value formulation to generate an optimal motion with regard to the power consumption. Combinations of pre-recorded motion capture data and spacetime optimization was done in [9], [10], [11] to generate various kinds of human like motions. Another method to generate walking motions to use controllers that take feedback from a physical model to compute controls that keep the model in a desired motion. Hodgins et al. [12] used a finite state machine and proportional-derivative controllers to compute torques that generate the motion. In [13] low dimensional representations of kinematic recordings are selected by an optimization process which also considers user-specified constraints and physical validity into account. Probabilistic methods were used in [14] to combine statistical models that were obtained from motion capture data with physical models which allow the models to react to the environment. The controllers described in [15] learns controllers from reference data and [16] computes optimal nonlinear quadratic regulators using an offline process that takes motion capture data as an input.

But also without reference trajectories various methods exist to generate walking motions. E.g. the Simple Biped Locomotion Control framework (SIMBICON) originally described in [17] does not require any motion-capture input. It has been further developed by employing controller optimization in [18]. In [19] a robust controller for virtual humans is described that can be used for a wide range of bipedal models and styles. It combines multiple techniques.
such as an inverted pendulum model to achieve high stability and also allows climbing of stairs. A similar method using these task-space controllers is also described in [20].

More recently muscle models [21] have been used to create real-time controllers [22] of the lower-body that also reproduce kinematic and dynamic joint forces found in human data. In [23] this work is extended to further optimize the location and routing of the muscles for various bipedal characters. There have been many more methods that use physically-based human models and for a more complete overview we refer to [24]. There is further a lot of research being done to create controllers for quadruped gaits [25], [26].

In this paper we generate physically valid motions by using optimal control techniques, which are sometimes referred to as *spacetime constraints* [8] or trajectory optimization methods. Optimal control methods have the advantage that it is not required to prescribe exact trajectories or fixed keyframes for all degrees of freedom. Instead we only need to describe parts of the motion (e.g. intermediate postures, joint ranges, foot positions) and leave it to the optimization to find the best motion that fulfill these constraints for a simulated model. The prescribed intermediate postures can be enforced but are otherwise free and are adjusted during the optimization. In this process both torques and trajectories of the joint angles are generated.

When dealing with human walking and optimal control methods one has to take special care of the discontinuities arising from foot contact gains. The approach presented in [27] avoids these problems by smoothing out the contacts [28] by introducing penalty terms in the objective function that may suffer residual forces that are not completely optimized away. The approach presented in this paper includes the discontinuities arising from foot contact gains and uses a prescribed set of contact constraints and thus avoids residual forces. The approach has already been successfully used to synthesize human running [29], [30] and 2-D walking [2] without the use of motion capture data.

In this paper we first describe the gait we consider and how we model the rigid multibody dynamics as a piecewise continuously differentiable process, which allows it to be used in an optimal control context. We then describe the optimal control problem that describes an optimal gait and the method we used to solve it. We present different objective functions that we used to generate different motions and compare those with reference motion capture data.

III. HUMAN GAIT MODELING

A. Phases of Gait

We model the human gait as a sequence of multiple phases. On each phase different contact constraints act on the feet of the human model. In this paper we consider a single step which starts when the right foot is flat on the ground and the left foot just started to swing and the step ends when the right foot lifts off the ground.

In total we have five phases: Right Flat, Right Forefoot, Right Forefoot Left Heel and Right Forefoot Left Flat. Additionally we model touch down events of the left heel and left forefoot as discontinuous events. The phases of the step are visualized in Figure 2. Even though we only consider a single step here one should note that it is not a limitation of our model but instead can be extended to a double or a multi-step sequence. Focusing on a single step however leads to fewer variables in the gait synthesis optimal control problem. We further introduce gait parameters $p_g = \{p_l, p_v\}$ where $p_l$ is the step length and $p_v$ the walking velocity. The parameters can be either prescribed or left free for the optimization.

B. Human Multibody Model

We use articulated rigid multibody models to formulate the dynamics of the full-body human model. For the dynamics modeling we created RBDL - the Rigid Body Dynamics Library which uses a reduced coordinate approach to model kinematics and dynamics of multibody systems. It is an
open-source implementation [31] of various state-of-the-art reduced coordinate algorithms for multibody dynamics described in [32].

The generalized positions that define the state of the system at time $t$ is described by $q(t) \in \mathbb{R}^{n_{\text{dof}}}$, where $n_{\text{dof}} \in \mathbb{N}$ are the numbers of degrees of freedom of the model. Further we denote with $\dot{q}(t), \ddot{q}(t), \tau(t)$ the generalized velocities, accelerations, and forces acting on the rigid multibody system. In the following sections we omit the parameter $t$ for brevity.

To model the human we use biomechanical data [33] for kinematic and inertial properties (i.e. locations of joints and mass, center of mass, and inertia for each body). As for the degrees of freedom we use six for the pelvis, three for the hips, one for the knees, three for the ankle, three for the lumbar joint, three for the shoulders, one for the elbow and three for the neck, which results in a total of 34 DOFs for the human model.

C. Constrained Rigid Multibody Dynamics

During walking at least one foot is in contact with the ground, which gives rise to external constraints on the rigid multibody model. A rigid multibody model with $m$ external constraints is described by the following differential algebraic equation of (differential) index 3:

$$
\begin{align}
H(q)\ddot{q} + C(q, \dot{q}) &= \tau + G(q)^T \lambda, \\
g(q) &= 0,
\end{align}
$$

(1a)

(1b)

where $H(q)$ is the joint space inertia matrix, $C(q, \dot{q})$ are the Coriolis forces, $G(q) = \frac{\partial}{\partial q} q$ is the so-called contact Jacobian matrix of size $m \times n_{\text{dof}}$ and $\lambda \in \mathbb{R}^m$ are the contact forces. By differentiating the constraint equation (1b) twice we obtain:

$$
\begin{align}
H(q)\dddot{q} + C(q, \ddot{q}) &= \tau + G(q)^T \dot{\lambda}, \\
G(q)\dot{\ddot{q}} + \dot{G}(q)\dot{q} &= 0,
\end{align}
$$

(2a)

(2b)

which we can rewrite as a linear system of the unknowns $\dot{q}, \dot{\lambda}$:

$$
\begin{bmatrix}
H(q) & G(q)^T \\
G(q) & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{\lambda}
\end{bmatrix}
= \begin{bmatrix}
-C(q, \dot{q}) + \tau \\
\gamma(q, \dot{q})
\end{bmatrix}
$$

(3)

This system is always solvable if $G(q)$ has full rank, which is the case if the constraints in $g(q)$ are not redundant. The term $\gamma(q, \dot{q}) \in \mathbb{R}^m$ is the negative right summand of (2b) and is also called contact Hessian.

To ensure equivalence of (1) and (3) we have pose constraints that the invariants of the constraints are fulfilled:

$$
\begin{align}
g(q) &= 0, \\
G(q)\dot{q} &= 0.
\end{align}
$$

(4)

(5)

It is sufficient to pose these conditions only at the beginning of the contact as due to (2b) and therefore (3) the constraint conditions are already fulfilled on the acceleration level.

The solution of (3) can be embedded in a ordinary differential equation as

$$
\dot{x} = f_1(x(t), u(t))
$$

(6)

where $x(t) = [q(t)^T, \dot{q}(t)^T]^T$ and $u(t)$ are the joint forces of the actuated joints that get mapped onto the generalized forces via a selection matrix $T$ by writing $\tau = Tu$. As different set of contact constraints lead to different algebraic constraints we use $i$ to indicate the currently used constraint set.

When using numerical integration equation (2b) will not be fulfilled exactly, hence errors will accumulate eventually. This is especially true for large step sizes and/or long simulation durations. In this case one can use Baumgarte stabilization [34]. In this work however no stabilization was used as the integration horizons are relatively short and no large error accumulations have been observed.

1) Collision Impacts: The transition from a rigid-body system without contacts to a system that has contacts is called a contact gain. During a contact gain very high forces act on the body for a very short time. In the real world these high forces cause the body to first compress and then expand. After the compression phase, depending on the physical properties of the body, the body remains either in contact (perfect inelastic collision) or bounces off.

In rigid-body simulations the compression and expansion is usually neglected and the contact gain is treated as an instantaneous collision. The physical properties of the body are described by the parameter of restitution $e \in [0, 1]$. For $e = 0$ the collision is a perfect inelastic collision, whereas for $e = 1$ the collision is perfectly elastic.

The contact gain is a discontinuous change in the generalized velocity variables from $\dot{q}^{-}$ to $\dot{q}^{+}$, i.e. the velocity before the collision to the velocity after the collision. The change can be computed using

$$
\begin{bmatrix}
H(q) \\
G(q)
\end{bmatrix}
\begin{bmatrix}
\dot{q}^{+} \\
\dot{q}^{-}
\end{bmatrix}
= \begin{bmatrix}
0 \\
-A
\end{bmatrix}
$$

(7)

which is the change of momentum of the system due to the collision. The lower part of this equation implies

$$
H(q)\dot{q}^{+} - H(q)\dot{q}^{-} = G(q)^T A,
$$

(8)

where $A$ is the contact impulse. The upper part of this equation states the contact velocity after the collision. For $e = 0$ this is equivalent to a contact velocity of zero, a perfect inelastic collision.

Similar to (6) one can embed the transition from before the contact to after the contact as

$$
x(t_i^+) = c_i(x(t_i^-)).
$$

(10)

2) Efficient Solution of the Contact Systems: The contact systems (3) and (7) have the same matrix on the left-hand side which is regular if $G(q)$ has full rank. In this section we describe the numerical solution of (3), however the same method can be used to solve (7).
There are different ways of solving these systems. By setting \( e(q, \dot{q}, \tau) = -C(q, \dot{q}) + \tau \) and omitting the arguments \( q, \dot{q}, \) and \( \tau \) we can write the system as

\[
\begin{bmatrix}
H & G^T \\
G & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
-\lambda
\end{bmatrix} =
\begin{bmatrix}
c \\
\gamma
\end{bmatrix},
\tag{11}
\]

where \( H \in \mathbb{R}^{n_{\text{act}} \times n_{\text{act}}} \), \( G \in \mathbb{R}^{m \times n_{\text{act}}} \), and with \( 0 < m < n_{\text{act}} \) being the number of contact constraints.

The matrix in the linear system is of the same structure as the so-called Karush-Kuhn-Tucker (KKT) matrix that plays an important role in quadratic optimization problems. The matrix is regular if \( G \) has full rank and is furthermore indefinite [35].

The matrix in (11) is symmetric but indefinite. An efficient method to solve the system is the symmetric indefinite \( LDL^T \) factorization. Nevertheless more efficient methods are available that exploit the structure of the matrix.

The so-called range-space method (or sometimes called Schur-complement method) first solves for \( \lambda \) and then \( \dot{q} \).

The former is computed using

\[
GH^{-1}G^T \lambda = \gamma - GH^{-1}c
\tag{12}
\]

and the latter is obtained by solving for \( \dot{q} \) using the system

\[
H \ddot{q} = c + G^T \lambda
\tag{13}
\]

(12) can be obtained by multiplying the upper part of (11) with \( GH^{-1} \) and then subtracting the result from the bottom part.

The matrices \( GH^{-1}G^T \) and \( H \) are both symmetric and positive definite which allows us to use a Cholesky decomposition to solve the equations (12) and (13). It suffices to factorize \( H \) just once and use the resulting factors to compute the columns of \( H^{-1} G^T \) and the vector \( H^{-1} c \) in (12) and additionally when solving (13) to compute the accelerations. As \( H(q) \) has a sparse structure for branched system one can also exploit these so-called branch-induced sparsities to derive even more efficient methods that are described in [36] and which we have also implemented in RBDL.

D. Ground Contact Modeling

To model the interaction of the model with the ground we developed a new rigid-body contact model shown in Figure 3. The sphere model to roll over the heel similarly as for a real human foot during early stance phase. Once the line segment at the forefoot is in contact with the ground the foot cannot move anymore. The heel starts to lift lifts as soon as the vertical ground reaction force vanishes and the foot can rotate around the axis defined by the line segment.

E. Actuation

To model passive elements such as tendons and ligaments we add spring-damper elements on all actuated degrees of freedom. Let \( n_a \in \mathbb{N} \) denote the number of actuated degrees of freedom (in our case \( n_a = n_{\text{act}} - 6 \)). The force produced by the passive spring-damper element acting on the degree of freedom with index \( i \) can be stated as:

\[
\tau_{sd,i}(t) = k_i(q^0_i - q_i(t)) - b_i(\dot{q}_i(t)) \in \mathbb{R},
\tag{14}
\]

where \( k_i > 0 \in \mathbb{R} \) is the spring constant, \( q^0_i \) is the rest position of the spring, \( q_i(t) \) is the position of the joint, \( b_i > 0 \in \mathbb{R} \) is the damping constant, and \( \dot{q}_i(t) \) is the joint velocity. For joints that do not have a spring-damper we set \( \tau_{sd,i}(t) = 0 \). By assembling all spring-damper parameters into a single parameter vector \( p_{sd} = (k_1, q^0_1, b_1, k_2, q^0_2, b_2, ..., k_{n_a}, q^0_{n_a}, b_{n_a})^T \) allows us to write the vector of all passive action forces as \( \tau_{sd}(q(t), \dot{q}(t), p_{sd}) \in \mathbb{R}^{n_{\text{act}}} \). The complete actuation can then be written as:

\[
\tau(t) = T u(t) + \tau_{sd}(q(t), \dot{q}(t), p_{sd}).
\tag{15}
\]

IV. GAIT SYNTHESIS OPTIMAL CONTROL FORMULATION

In our optimal control formulation we solve for the optimal state trajectories \( x(t) = [q(t), \dot{q}(t)]^T \), joint actuation trajectories \( u(t) \), phase durations \( t_1, ..., t_4 \), and gait and spring-damper parameters \( p \).

Formally the gait synthesis optimal control problem can be stated as:

\[
\begin{align*}
\min_{x(\cdot), u(\cdot), t_1, ..., t_4, p} & \quad \Phi(x(t), u(t), p) \\
\text{subject to:} & \quad \dot{x}(t) = f(x(t), u(t), p), \quad t \in [t_{i-1}, t_i], \\
& \quad x(t_i^+) = c_i(x(t_i^-)), \quad i \in \mathbb{Z}, \\
& \quad g_i(x(t), u(t), p) \geq 0, \quad t \in [t_{i-1}, t_i], \quad i \in \mathbb{Z}, \\
& \quad r(x(t_1), ..., x(t_k), u(t_1), ..., u(t_k), p) \geq 0, \quad t_1, ..., t_k, \in \mathbb{T}, \quad \mathbb{Z}.
\end{align*}
\tag{16a}
\]

Eq. (16a) is the objective function that we describe in the following subsection in detail.

The dynamics of the model during the different phases are described using the ordinary differential equations (16b),
The individual right-hand side functions \( f_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_x} \) describe the different dynamics subject to the different constraint sets and are influenced by the controls \( \mathbf{u} : \mathcal{T} \to \mathbb{R}^{n_u} \).

Phase transitions, specifically collisions at touch-down events of the foot with the ground, are described using the phase transition functions \( c_i(\mathbf{x}(t)) \). The collisions are evaluated as described in Section III-C.1 with at restituation parameter \( e = 0 \).

The path constraints (16d) contain upper and lower bounds for the differential states \( \mathbf{x}(t) \) and controls \( \mathbf{u}(t) \). The bounds for \( \dot{\mathbf{q}}(t) \) are obtained from motion capture data of neutral human walking by using the upper and lower bounds across multiple trials and an offset by a margin such that the optimized motion is not constrained by these bounds in the final solution. For the generalized velocities \( \dot{\mathbf{q}}(t) \) we use the bounds \(-15.0 \leq \dot{q}_i(t) \leq 15.0, i = 1, \ldots, n_{\text{dof}}\). For the controls \( \mathbf{u}(t) \) we chose bounds such that they were large enough to not affect the solution.

The general point constraints (16e) formulate additional constraints such as foot clearance, proper ground reaction forces (i.e. that the model does not get pulled by the ground), and ensure that segments, approximated by capsules, do not intersect.

### A. Objective Functions

We have formulated multiple objective criteria that are used to formulate the composite objective function

\[
\Phi(\mathbf{x}(t), \mathbf{u}(t), p) = \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} \left( \sum_{j=1}^{3} \gamma_j \Phi_{L_j}(t, \mathbf{x}(t), \mathbf{u}(t), p) \right) dt + \gamma_4 \Phi_{M_4}(t_f, \mathbf{x}(t_f), \mathbf{p}).
\]

(17)

The weighting factors \( \gamma_j \in \mathbb{R}, j = 1, \ldots, 4 \) can be adjusted to obtain different weightings of the basic objective criteria. The basic objective criteria are:

a) **Minimize Active Actuation over Step Length**: As a measure for the required effort to produce a step we use the objective function

\[
\Phi_{L_1}(t, \mathbf{x}(t), \mathbf{u}(t), p) = \frac{1}{p_t n_u} || \mathbf{W} \mathbf{u}(t) ||^2_2,
\]

(18)

which minimizes the weighted active actuation normalized by the step length \( p_t \) and the number of controls \( n_u \).

The diagonal matrix \( \mathbf{W} \in \mathbb{R}^{n_x \times n_x} \) contains weightings that normalize the different joint contributions. We used the weightings listed in Table I, that we obtained by reconstructing the joint actuation from motion capture data and taking their averages over a step cycle [37]. The same weightings are used for the joint actuations on left and right side of the body.

b) **Minimize Angular Momentum**: The system angular momentum describes the rotational momentum that the rigid-body system has about a certain point. In our case we use the system angular momentum expressed at the center of mass. The objective function can be written as:

\[
\Phi_{L_2}(t, \mathbf{x}(t), \mathbf{u}(t), p) = || \mathbf{m}_{CM}(\mathbf{q}(t), \mathbf{q}(t)) ||^2_2,
\]

(19)

where \( \mathbf{m}_{CM}(\mathbf{q}(t), \mathbf{q}(t)) \) is said quantity.

c) **Minimize Head Angular Velocities**: To formulate some kind of head stabilization criterion we formulated the objective function:

\[
\Phi_{L_3}(t, \mathbf{x}(t), \mathbf{u}(t), p) = || \mathbf{E}_{\text{head}} \mathbf{\omega}_{\text{head}} ||^2_2.
\]

(20)

Here \( \mathbf{E}_{\text{head}} \mathbf{\omega}_{\text{head}} \) is the orientation of the head segment relative to the global reference frame its product with the angular velocity of the head \( \mathbf{\omega}_{\text{head}} \) results in the angular velocity of the head segment in the global reference frame. 0.

d) **Minimize Step Time**: The objective function that minimizes the duration of the step (not to be confused with the walk velocity) is stated as a Mayer term:

\[
\Phi_{M_4}(t_f, \mathbf{x}(t_f), p) = t_f.
\]

(21)

It minimizes the time it takes to complete a single step.

We chose weightings \( \gamma_j \) to formulate the following objective functions from the basic objective functions:

<table>
<thead>
<tr>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3 )</th>
<th>( \gamma_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_1 )</td>
<td>( \Phi_2 )</td>
<td>( \Phi_3 )</td>
<td>( \Phi_4 )</td>
</tr>
<tr>
<td>( \text{Min. Act. over Step Length} )</td>
<td>( \text{Min. Ang. Momentum} )</td>
<td>( \text{Min. Head Ang. Vel.} )</td>
<td>( \text{Min. Step Duration} )</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.01</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0</td>
<td>10.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0</td>
<td>0.0</td>
<td>10.0</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>3.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Each of \( \Phi_1, \ldots, \Phi_4 \) mainly minimizes a single basic objective criteria and a small weighting of the of the \( \Phi_{L_1} \) criterion to regularize the optimal control problem. Without this regularization the controls may take arbitrary values without affecting the actual solution. Additionally we formulated the multi objective objective function to see how multiple criteria affect the solution. The weightings of 10.0 in \( \Phi_3 \) and \( \Phi_4 \) is used to account for the different orders of magnitude of the basic objective functions.

<table>
<thead>
<tr>
<th>Joint (Axis)</th>
<th>( w^{-1}_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip (Y)</td>
<td>15.5</td>
</tr>
<tr>
<td>Hip (X)</td>
<td>18.0</td>
</tr>
<tr>
<td>Hip (Z)</td>
<td>3.6</td>
</tr>
<tr>
<td>Knee (Y)</td>
<td>14.4</td>
</tr>
<tr>
<td>Knee (Z)</td>
<td>39.4</td>
</tr>
<tr>
<td>Ankle (Y)</td>
<td>5.2</td>
</tr>
<tr>
<td>Ankle (Z)</td>
<td>39.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Joint (Axis)</th>
<th>( w^{-1}_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ankle (X)</td>
<td>2.5</td>
</tr>
<tr>
<td>Shoulder (Y)</td>
<td>19.0</td>
</tr>
<tr>
<td>Shoulder (Z)</td>
<td>2.5</td>
</tr>
<tr>
<td>Neck (Y)</td>
<td>13.9</td>
</tr>
<tr>
<td>Neck (Z)</td>
<td>1.5</td>
</tr>
<tr>
<td>Shoulder (X)</td>
<td>15.0</td>
</tr>
</tbody>
</table>

**Table I**

WEIGHTING COEFFICIENTS TO ACCOUNT FOR THE DIFFERENT JOINT STRENGTHS IN THE REGULARIZATION TERM OF (18).

### B. Numerical Solution of the Optimal Control Problem

To solve the optimal control problem (16) we use a direct multiple shooting method [38] which is implemented in the software package MUSCOD-II. It discretizes the continuous formulation (16a)–(16e) for both controls and states by dividing the time horizon in \( M \) so-called multiple shooting intervals. The state trajectories are parameterized as starting values \( s_j \) for initial value problems defined for each
multiple shooting interval $j$. The controls are discretized by parameters $u_j$ for simple base functions on each multiple-shooting interval, such as piecewise constant, piecewise linear or spline functions for each interval. To ensure that the resulting state trajectories represent a continuous solution additional continuity conditions
\[ x(t_{j+1}; s_j, u_j) - s_{j+1} = 0 \]
are formulated.

By doing so the functions $x(t)$ and $u(t)$ were replaced by their finite dimensional counterparts $s_0, \ldots, s_M$ and $u_0, \ldots, u_{M-1}$. Further discretization of the constraints and objective function leads to an nonlinear optimization problem of the form:

\[ \min_y F(y) \]

\[ \text{subject to:} \]
\[ g(y) \geq 0 \]
\[ h(y) = 0 \]

This problem is then solved by using a specially tailored sequential quadratic programming (SQP) method that exploits structures arising from the direct multiple-shooting approach.

V. RESULTS

For our human gait optimal control problem formulation we used 16 multiple shooting nodes and piecewise linear base functions for the control discretizations. Overall the discretized optimal control problem problem has 2381 free variables, 1887 equality and 4885 inequality constraints. In Figure 1 we show the synthesized gait for the multi objective criterion $\Phi_5$ and refer to the supplementary video material for the other gaits.

A. Comparison of Synthesized and Recorded Motions.

To compare the synthesized gaits $q^S(t) : [0, t_f^S] \rightarrow \mathbb{R}^{n_{q^S}}$ with the recorded motion capture data $q^M(t) : [0, t_f^M] \rightarrow \mathbb{R}^{n_{q^M}}$ we use the following similarity measure to compare two motions:

\[ d(q^S(t), q^M(t), t_f^S, t_f^M) = \]
\[ \frac{1}{2} \sum_{i=1}^{m} \left( \frac{1}{m \cdot n_{\text{def}}} ||q^S_i(i \Delta t^S) - q^M_i(i \Delta t^M)||^2_2 \right) \]
\[ + \frac{1}{2} \left( t_f^S - t_f^M \right)^2, \quad (22) \]

with $\Delta t^S = t_f^S/(m - 1)$ and $\Delta t^M = t_f^M/(m - 1)$. The similarity measure evaluates the difference of the two motions in terms of the postural error, i.e. the least-square error of the generalized joint positions on an equidistant time grid, and the temporal error in terms of the difference of the step durations. The postural error normalized both by the number of evaluations $m$ and the number of degrees of freedom $n_{\text{def}}$.

To evaluate the similarity of the synthesized gaits with the measured movements we computed a reference motion by averaging the time-normalized joint trajectories $q(t)$ of three recordings of a neutral gaits of a single subject. Similarly we computed an averaged step duration that we use as the reference duration.

1) Similarity of the Synthesized Motions and Motion Capture Data: We compared the similarity of the reference motion with synthesized gaits obtained by variation of the objective function. The results are shown in Figure 4 for an evaluation of $m = 100$.

The objective function Multi Objective has best similarity value, followed by Min. Act. over Step Length and Min. Ang. Momentum. The worst is Min. Step Duration, which is however due to the large contribution of the $t_f$ term. The solution of Min. Head Ang. Velocity shows the smallest error in the least-squares term of the joint angles.

Overall the use of multiple criteria yielded a slight improvement in both postural and temporal similarity. This could be interpreted that human gait is governed by multiple optimality criteria.

VI. CONCLUSION AND OUTLOOK

We have presented a optimal control approach for the synthesis of full-body 3-D human-like walking. We formulated the human gait as a piecewise continuously differentiable process suitable for the optimal control method employed in this paper, and also how we formulate the multibody dynamics of the human model. We presented the optimal control formulation and four basic objective functions and a multi-objective criterion for which we synthesized human walking motions. To assess the effect of the different optimization criteria we formulated a similarity measure and compared the synthesized gaits with motion capture data.

In the future we would like to extend the gait formulation to allow more movements, such as turning, climbing stars, and slopes. Also the transitions between walking and running would be of interest. Long term goals are to use our approach to identify optimality criteria of emotional human walking. These so-called inverse optimal control problems [39] further add challenging aspects to the numerical solutions, however new methods have been proposed recently [40].
REFERENCES


[38] Bock1984.pdf
